Composites with tuned effective magnetic permeability

Alireza V. Amirkhizi and Sia Nemat-Nasser[a)]
Center of Excellence for Advanced Materials, Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, California 92093-0416

(Received 6 November 2006; accepted 17 May 2007; published online 2 July 2007)

Pendry *et al.* [J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999)] and Smith *et al.* [D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000)] have shown that the effective magnetic permeability, μ, of free space can be rendered negative over a certain frequency range by a periodic arrangement of very thin conductors with suitable magnetic resonance properties, the so-called split-ring resonators. Because of its rather bulky architecture, this structure does not lend itself to a proper integration into a reasonably thin real composite structural panel. To remedy this fundamental barrier, we invented a new magnetic resonator consisting of very thin folded plates that are suitably nested within one another to form folded-doubled resonators (FDRs) that can be integrated into an actual composite panel. Measurements, using a focused beam electromagnetic characterization system combined with time-domain numerical simulations of the reflection and transmission coefficients of such a composite slab have revealed that indeed the composite has a negative μ over a frequency range of about 9.1–9.35 GHz [S. Nemat-Nasser, S. C. Nemat-Nasser, T. A. Plaisted, A. Starr, and A. Vakil Amirkhizi, in *Biomimetics: Biologically Inspired Technologies*, edited by Y. Bar Cohen (CRC Press, Boca Raton, FL, 2006)]. Thus, it has become possible to construct a structural composite panel with negative index of refraction by simultaneously creating negative effective ε and μ [V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968); R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001); A. F. Starr, P. M. Rye, D. R. Smith, and S. Nemat-Nasser, Phys. Rev. B **70**, 113102 (2004)].
[DOI: 10.1063/1.2751084]

I. INTRODUCTION

The overall electric permittivity of composites is modified by inclusion of arrays of certain conductive components such as thin straight wires or coils.[1–4] The influence of the scattering elements on the propagation of electromagnetic waves in a composite modifies its overall (effective) dielectric constant. In most cases, this is achieved by arrays of conductors that create electric dipoles in response to the oscillating electric fields; see Fig. 1. The properties of the electric dipole array determine the overall response of the material. For example, the overall inductance of the medium dictates the characteristic resonance frequency of the unit cell and therefore the dispersive behavior of the composite.

Similarly, in order to modify the magnetic properties of a composite, one can embed scattering structures within a composite such that an oscillating magnetic field would create current distributions that produce a magnetic dipole resonator. A simple example is a resonating current loop. Pendry *et al.*[1] studied various examples of such structures in some detail. They predicted that, using certain designs, one can render the overall magnetic permeability of the composite negative. Smith *et al.*[2] experimentally verified these predictions by fabricating one of the structures suggested by Pendry,[1] and measuring its transmission spectrum.

All such examples have the basic property of being based on resonating current loops. Another important aspect is that all of the current loops have a gap, since, in the absence of a gap, the restraining electromotive force against the induced current (assuming perfect conductors) would be rather small. On the other hand, when a gap is introduced in the loop, the induced current charges a capacitor that in turn creates an opposing electromotive force. Pendry *et al.*[1] studied simple loops and concluded that they have a very small magnetic effect.

While the electric dipoles define the effective inductance of the medium, it is the effective capacitance that, along with the overall inductance and the geometry of the scattering array, controls the magnetic properties of the composite. The original split-ring resonators increase the capacitance of the medium by having two separate resonators that were ar-

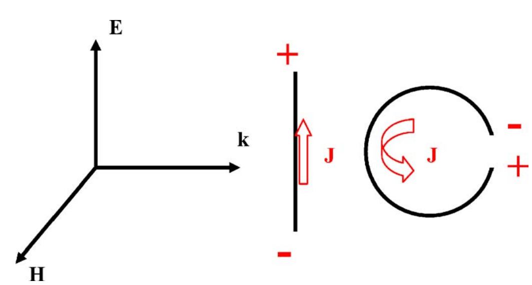

FIG. 1. Electric and magnetic dipole resonators. (left) Wave polarization. (center) An electric dipole resonator induced in a straight conductor and the accompanying current density. (right) A magnetic dipole resonator induced in a conductive split ring and the accompanying current density.

a)Author to whom correspondence should be addressed; present address: 9500 Gilman Dr., La Jolla, CA 92093-0416; electronic mail: sia@ucsd.edu.

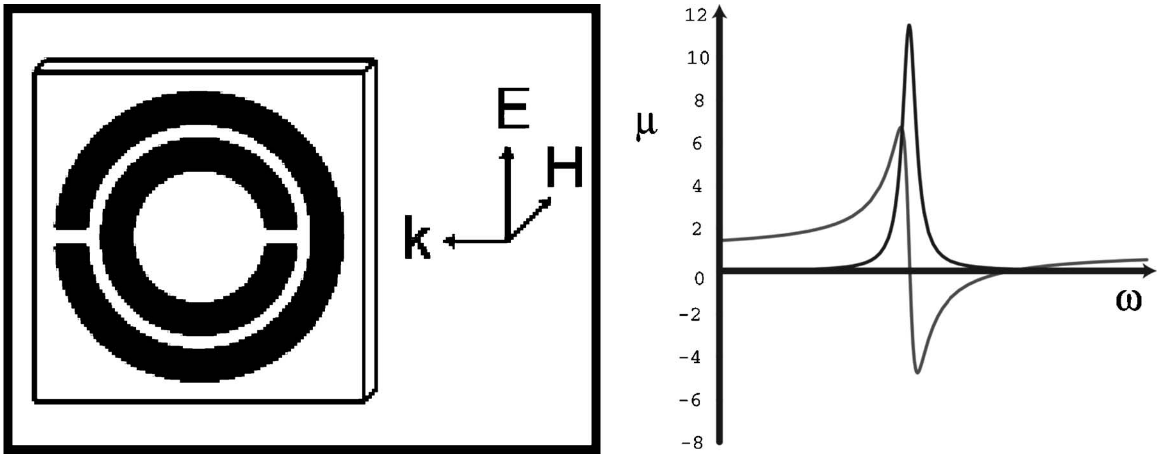

FIG. 2. Split-ring resonators. (left) Schematics of a split-ring resonator with the wave polarization that can excite it. (right) A typical dispersion curve for effective magnetic permeability normalized with respect to μ_0. Note that the real permeability becomes negative but the imaginary part is always positive as expected for a passive medium. (Drawing and graph courtesy of D. R. Smith.[10])

ranged to constitute the two surfaces of a capacitor. As shown by Starr *et al.*,[3] one may achieve a similar result using single split rings that are modified to have sufficient capacitance to suitably reduce the effective magnetic permeability of the composite.

II. SPLIT-RING RESONATORS

The first structure that was extensively studied and showed a considerable effect on the overall magnetic response of the medium consisted of split-ring resonators. It was introduced and analytically studied by Pendry *et al.*[1] and later fabricated and tested by Smith *et al.*[2] Its basic elements and a typical dispersion curve for its normalized overall magnetic permeability are shown in Fig. 2. As is seen, the real part of the magnetic permeability becomes negative but its imaginary part is always positive, as expected from a passive medium.

Pendry *et al.*[1] gave an analytical prediction for the effective magnetic permeability based on the line and surface integral averaging method.[5,6] A somewhat modified formula, used by Smith *et al.*,[2] is

$$\frac{\mu(\omega)}{\mu_0} = 1 - \frac{\omega_0^2 F}{\omega^2 - \omega_0^2 + i\Gamma\omega}. \quad (1)$$

Here, F is a geometric quantity that can be interpreted as the filling factor; Γ is a loss parameter that depends on the conductivity of the rings; and ω_0 is the resonance frequency which depends on the capacitance of the element. These parameters can be calculated based on the geometry of the unit cell and the material properties; for details, see Ref. 2.

The index of refraction, $n = \sqrt{\mu\varepsilon}$, becomes imaginary in the negative magnetic permeability frequency band in which there are no propagating modes within the composite, the only possible modes being the evanescent waves. All the energy of a wave in this frequency regime will be either reflected or absorbed within a thin layer of the composite. Smith *et al.*[2] verified the existence of this stop band using a scalar network analyzer and measuring the transmitted power in a waveguide filled with a split-ring resonator array.

III. ORIGINAL FOLDED-DOUBLED RESONATORS

The split-ring resonator design is easily tunable and demonstrates a relatively wide band with negative effective permeability. Consequently, the corresponding permeability, $\mu(\omega)$, varies slowly with frequency within this band. It permits tuning within a wide-band frequency. However, the split-ring resonators cannot be easily integrated into a real structural composite of limited thickness, since the plane on which the split-rings rest must be perpendicular to the plane of the composite panel, parallel to the wave vector, to interact with the incident waves that are normal to the composite panel. Ideally, one would like to be able to stack layers of magnetic resonators between prepreg layers or fiber bundles. Therefore, it is important from the manufacturing point of view, to create unit cells that can be embedded in a fiber reinforced composite, compatible with its layered structure and amenable to common fabrication methods. One such de-

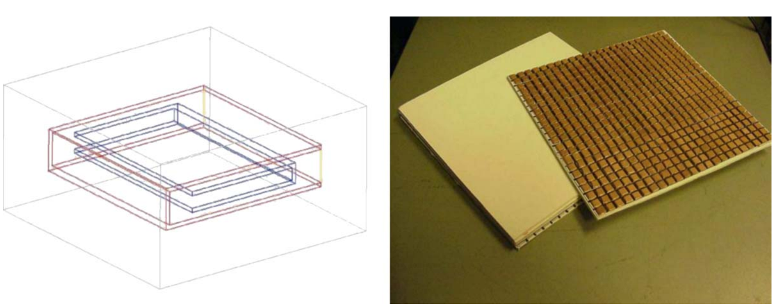

FIG. 3. (left) Schematics of a folded-double resonator. (right) Samples made from adhesive copper tape.

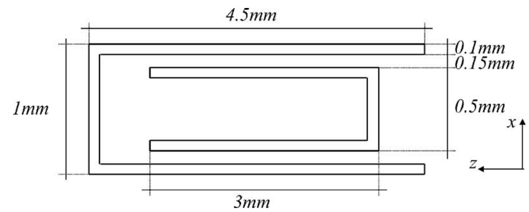

FIG. 4. Cross section of a FDR made from adhesive copper tape with dimensions.

sign is the folded-doubled resonator (FDR). It consists of two folded conductive thin strips with one tucked inside the other, as shown in Fig. 3 (left). The geometry of this construction is suitable to being embedded into a layered fiber reinforced composite. The electric capacitance is then provided by the two folded conductive strips.

To examine this idea we fabricated samples using thin adhesive copper tapes. A 6.4 mm wide tape is folded and placed inside another tape of 10 mm width. The outer tape is then folded over. The adhesive backing of the copper provides the necessary insulation. It also modifies the capacitance of the resulting element. The copper thickness is about 0.1 mm and the total thickness of the FDR is about 1 mm; see Fig. 4. The folded strips are cut in 4.5 mm sections, giving planar cross sections of 4.5 mm by 4.5 mm and 4.5 mm by 3 mm outer and inner resonators, respectively. They are then glued on a thin Styrofoam panel and arranged into an array of 6 mm by 6 mm by 3 mm unit cells.

We have numerically simulated this structure using Ansoft-HFSS finite-element electromagnetic software.[7] The simulations are performed in the frequency domain, using a unit cell with periodic boundary conditions on its y and z faces such that

$$\mathbf{F}(x=x^{+}, \ y, \ z)=\mathbf{F}(x=x^{-}, \ y, \ z)e^{i\varphi}, \tag{2}$$

$$\mathbf{F}(x, \ y=y^{+}, \ z)=\mathbf{F}(x, \ y=y^{-}, \ z), \tag{3}$$

$$\mathbf{F}(x, \ y, \ z=z^{+})=\mathbf{F}(x, \ y, \ z=z^{-}), \tag{4}$$

$$\varphi=2\pi\frac{x^{+}-x^{-}}{\lambda}. \tag{5}$$

Here, **F** stands for any of the electromagnetic fields. The boundary condition for the x faces is periodic with a prescribed phase advance that is related to the wavelength through Eq. (5). A stop band between 6.09 and 6.51 GHz (negative permeability) is predicted by this calculation. The value of permeability was not directly calculated. However, the simulation shows that there are no propagating modes in this frequency range. Following the discussion in Pendry *et al.*,[1] we consider this as an indication that the overall index of refraction is imaginary. Since in this case the permittivity is positive, it follows that the effective permeability is negative.

The fabricated samples were tested in an anechoic chamber. The transmission through one, two, and three layers of this composite is measured in the 3–12 GHz range. The predicted stop band is verified. The depth of the stop band increase with the number of layers. The width of this band is considerably larger than the numerical predictions. This is partially due to imperfect hand fabrication technique. The FDRs in this sample are not uniform and the deviation from the perfect geometry (assumed in simulation) has affected the resonance frequency of the unit cells, as is seen from Fig. 5. Also shown in Fig. 5 is the transmitted power when the incoming wave is polarized in such a way not to interact with the FDRs. The FDR resonates when the incoming magnetic field is polarized along the y axis, but it is inactive when the magnetic field is polarized parallel to the z axis. The presence of the FDR affects the magnetic field in the entire unit cell, but its effect is greater within the volume enclosed by the plates. In certain cases, the direction of the magnetic field within this volume is opposite to that outside of this region, due to the strong induced currents in the FDR.

IV. *X*-BAND FDR COMPOSITES

The samples described in the previous section have two shortcomings. First, they are not structural composites. They have very low load-bearing strength or stiffness. Second, the samples have significant imperfections because of the fabrication process. To address these issues, we employed a conventional commercial printed circuit-board fabrication technology; see Fig. 6. The dimensions in the thickness

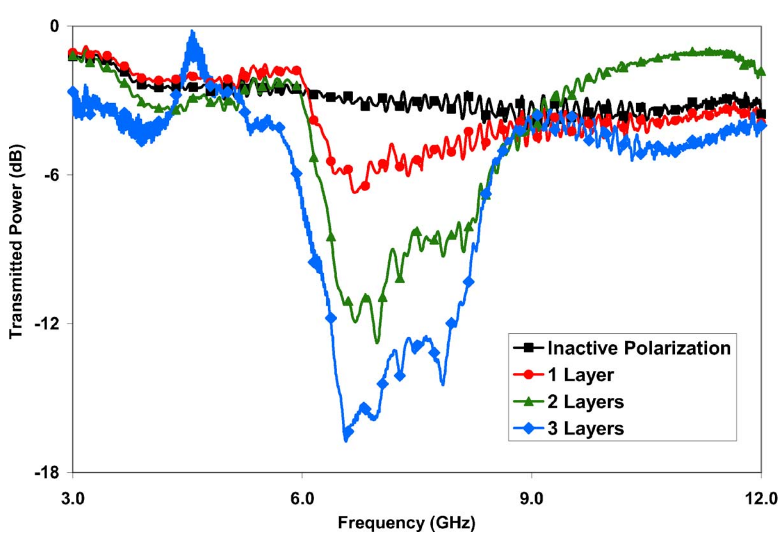

FIG. 5. The transmitted power through one, two, and three layers of the hand-fabricated FDR. Also shown, is the transmission when incident wave is polarized in such a way not to interact with the FDRs.

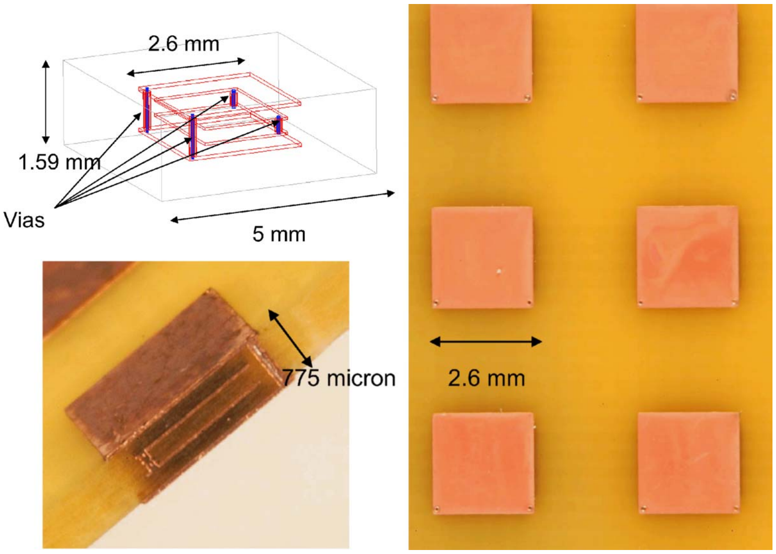

FIG. 6. *X*-band FDR. (top left) Schematic and dimensions. (bottom left) Cross section of the finished board. (right) Planar view of the board.

directions were selected based on the available material for printed circuit board (PCB) fabrication. The choice of other geometrical parameters was *ad hoc*; however, the dimensions were adjusted after a few preliminary numerical simulations. The plates of the inner resonator have a rectangular form, while the outer plates are square.

In this method, the inside resonator is made out of a single board of FR-4 material that is plated on both sides by copper. The copper is etched away into 2.6 mm by 1.9 mm rectangles to produce the inside resonators. Then, two vias, 160 μm nominal diameter, are drilled on one edge of the copper rectangles. The inside of the vias are plated with copper to provide electrical connection between the top and the bottom parts. The outside resonator is made out of two FR-4 boards that have copper on one face only. The outer resonator's platelets are 2.6 mm by 2.6 mm copper squares and are created similarly to the inner resonators. The finished inner board is sandwiched between the outer boards and treated under elevated pressure and temperature to make a single monolithic board. Finally, two vias are drilled and plated with copper to provide electrical connection between the outside copper squares. The average measured thickness of the final board is 775 μm. A more precise arrangement and spacing of the FDRs is ensured by this fabrication method.

To test this structure, a layered composite was created by stacking the printed FDR boards with 0.8 mm thick blanks in between to produce the unit cell with the designed dimensions. The resulting composite may be processed into a monolithic sample by elevated temperature and pressure treatment, but, for the purposes of the evaluation of its electromagnetic (EM) properties, it was sufficient to hold the layers together mechanically. In this manner, we were able to change the arrangement and the number of layers easily to examine the effect on the properties. The resulting structure was tested using a focused beam setup for electromagnetic characterization of composites, developed at UCSD's CEAM; see Fig. 7. The transmitted wave through the sample is measured using an Agilent 8510C vector network analyzer. The results are shown in Figs. 8 and 9. The curves shown represent the magnitude and phase of the transmitted wave through a five-layer slab of FDRs. The difference between the experimental curves is in the polarization of incident wave. For the solid curve the incident magnetic fields are polarized such that it creates a magnetic resonance in the FDRs; see Fig. 1. The dashed curve shows the transmission when the sample is rotated by 90 deg, that is when the incident wave is in inactive polarization, i.e., when the incident magnetic field does not excite resonance in the FDRs.

V. NUMERICAL SIMULATION AND PROPERTIES EVALUATION

The results of the transmission measurement were also modeled numerically. We simulated two structures; see Figs. 8 and 9. First, we assumed that all the material in the composite, except for the copper resonators, has a uniform dielectric constant of 3.6. Second, since the material of the spacing boards has different amount of resin and a processing history, we estimated and used a dielectric constant of 4.4

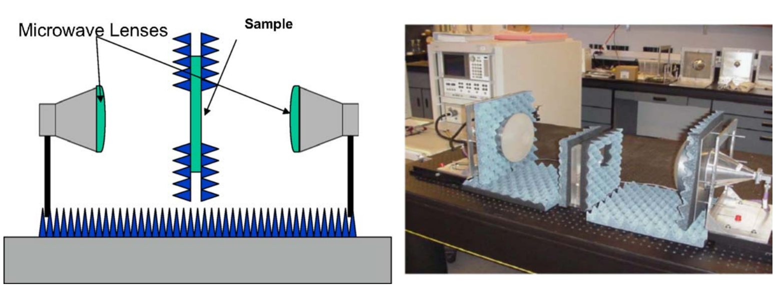

FIG. 7. UCSD/CEAM's focused beam setup.

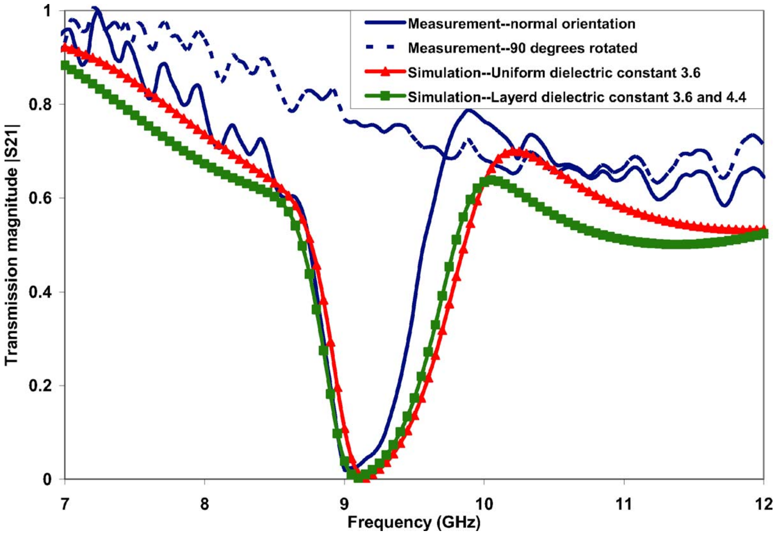

FIG. 8. Comparison between measured transmission (S_{21}) and simulated values of a five-layer printed board X-band FDR. The triangles are the simulation results when it is assumed that the entire composite (except for the conductors) has uniform dielectric constant of 3.6. The squares are the simulation results when layers of the printed board with dielectric constant 3.6 are sandwiched between FR-4 layers of dielectric constant 4.4. The solid and dashed lines are the measurements in the active and an inactive (obtained by 90 deg rotation) polarization, respectively.

for the boards that were sandwiched between the FDR layers. We simulated the structure shown in Fig. 10 and calculated the scattering parameters, S_{ij},

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}. \quad (6)$$

Here V_i^+ and V_i^- denote the voltage of the incident and scattered signals at the ith port, respectively. The results of the simulations are shown in Figs. 8 and 9.

To extract the overall material properties of the composite, we substitute the results of the simulation for S_{11} and S_{21} in the expected reflected and transmitted waves off of a slab of homogeneous material; see Fig. 11. Note that we chose to use the results of the numerical simulation with uniform dielectric constant 3.6, since it agrees with the experimental data very closely. Use of the simulated values has two advantages. First, we can avoid the noise of the measured data in a systematic way. The simulation results follow the experimental data closely but do not include various noise sources. Second, the reflection measurement, S_{11}, involves many internal reflections in the experimental setup. The value of the coefficient of reflection off of the sample, though not necessarily small, is dominated by the internal reflections in the measurement setup. Therefore, its measurement may be unreliable for most cases. The transmission measurement alone is not enough to yield the two complex-valued material parameters, n and z. This requires four real quantities, i.e., the magnitude and phase of both transmission and reflection coefficients. Therefore, we used both transmission and reflection data from simulation for this purpose. For a slab of thickness d, refractive index n, and impedance z, we have

$$t = \frac{1}{\cos(nkd) + \frac{i}{2}\left[z + \frac{1}{z}\right]\sin(nkd)}, \quad (7)$$

$$r = \frac{\frac{i}{2}\left[z - \frac{1}{z}\right]\sin(nkd)}{\cos(nkd) + \frac{i}{2}\left[z + \frac{1}{z}\right]\sin(nkd)}, \quad (8)$$

except for a phase factor $e^{\pm ikd}$. Here $k=\omega/c$ is the wave number for free space, t is the transmission coefficient, and r is the reflection coefficient; see Fig. 11. The inversion of these equations gives the overall material properties of the FDR composite. Here we followed the method of Smith

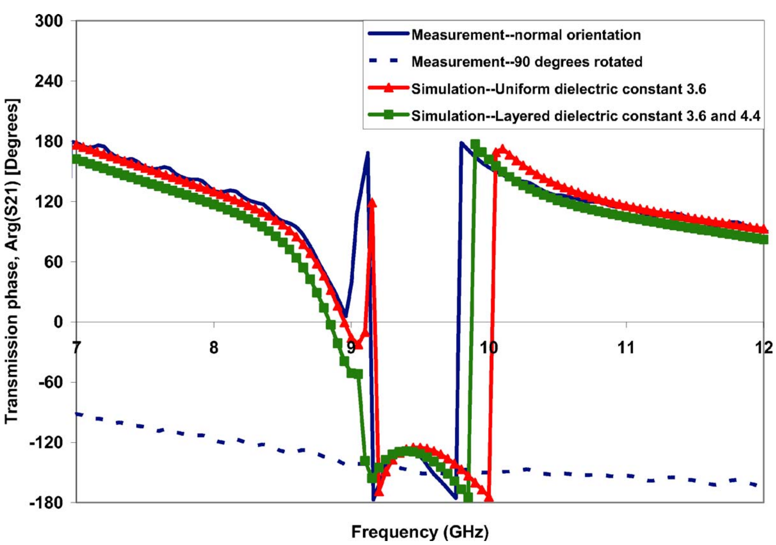

FIG. 9. Transmission phase for a five-layer printed board X-band FDR. Comparison between simulation and measurement results.

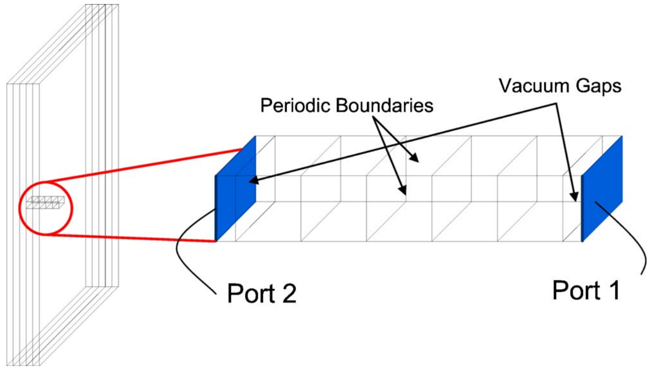

FIG. 10. Setup for time-domain simulation of a finite-thickness plate of the FDR composite. Each cube represents a unit cell.

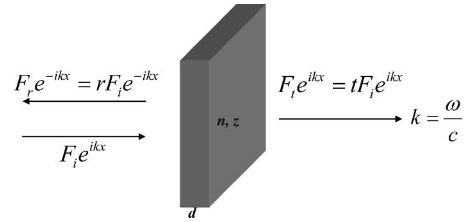

FIG. 11. Transmission and reflection of a monochromatic wave with wave number k off of a homogenous plate.

et al.[8] We used a MATLAB macro written originally by C. G. Parazzoli.[9] We adapted the macro for the current application by applying it to the output of the Ansoft-HFSS solver and de-embedding the results appropriately.

The material parameters calculated based on this inversion are given in Fig. 12. The results show a negative permeability band between 9.1 and 9.35 GHz, as well as an increase in the overall dielectric constant of about 7. The minimum normalized magnetic permeability is calculated to be around −0.65. We did not perform a sensitivity analysis for the material parameters n and z based on the alternate values of the dielectric constant in the unit cell. However, the numerical results, of the two cases we studied, agree with one another and with the experimental data.

VI. SUMMARY

We have designed a structural composite that exhibits negative overall magnetic permeability. Our work was distinguished from previous efforts in being the first to achieve structural integrity for a material with overall negative μ. We based our unit cell on the original idea of split-ring resonator by Pendry *et al.*[1] However, by introducing the folded-doubled resonator design, we were able to easily integrate this magnetic resonator into structural composites using conventional composite processing techniques. The FDR unit cell consists of two nested resonators. The flat shape of FDRs makes them a perfect candidate for embedding inside structural composites. The composite is fabricated from commercial lithography printed boards. The measured transmission through a five-layer slab of FDR indicates a stop band between 9.1 and 9.35 GHz. The numerical predictions are in good agreement with experiments. We used the results of numerical simulation and calculated overall material properties by inverting the transmission and reflection coefficients, using a homogeneous slab of comparable thickness. The calculated magnetic permeability shows a negative band between 9.1 and 9.35 GHz. This was the first fabrication and measurement of a structural composite with negative overall magnetic permeability. It led to a similar fabrication method that was subsequently used by Starr *et al.*[3] to produce a structural composite with negative index of refraction.

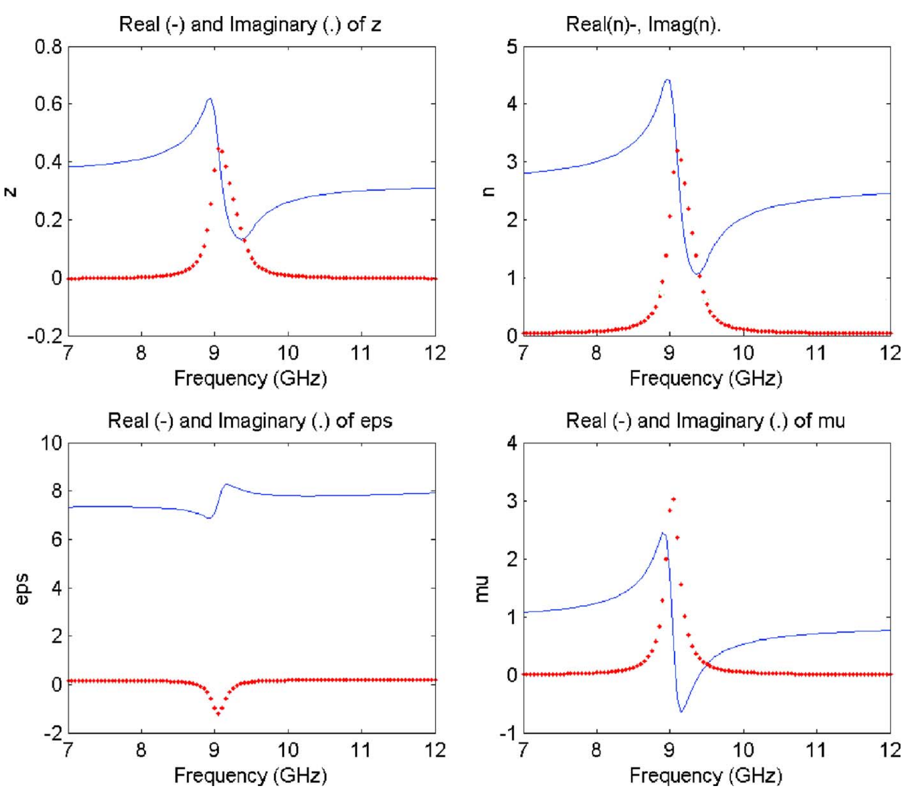

FIG. 12. Extraction of material properties of *X*-band printed board FDR from *S*-parameter simulation. Inversion macro, courtesy of Parazzoli (Ref. 9).

ACKNOWLEDGMENTS

The FDR idea was conceived by one of the authors (S.N.-N., October 2002), its simulation and design were completed by the first author (A.V.A.), and its EM measurements were performed by A. F. Starr (SENSORMETRIX, CA). The original version of the FDR was hand-constructed by D. Arbelaez (then an undergraduate, MAE student) with the help of T. Plaisted (graduate student) and CEAM technical staff, in early 2003. The *X*-band FDR composites were produced by Hughes Circuits, San Marcos, CA. It led to a new design by Starr *et al.*,[3] who coined the expression "The New Deal" for the new composite that had negative index of refraction in about 8.4–9.2 GHz frequency range. The program used for the inversion of the *S*-parameter data and extraction of material properties was kindly provided to the authors by C. G. Parazzoli and D. R. Smith and was adjusted for application to the present problem. This work was initially supported by DARPA through Grant. No. ARO DAAD19-00-1-0525 to the University of California, San Diego, and recently completed through partial support from AFOSR/MURI Grant No. FA9550-06-1-0337 to Kent State University; and Subaward No. 444286-PO61719 to University of California, San Diego.

[1] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).

[2] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).

[3] A. F. Starr, P. M. Rye, D. R. Smith, and S. Nemat-Nasser, Phys. Rev. B **70**, 113102 (2004).

[4] S. Nemat-Nasser, S. C. Nemat-Nasser, T. A. Plaisted, A. Starr, and A. Vakil Amirkhizi, in *Biomimetics: Biologically Inspired Technologies*, edited by Y. Bar Cohen (CRC Press, Boca Raton, FL, 2006).

[5] J. B. Pendry, J. Mod. Opt. **41**, 209 (1994).

[6] J. B. Pendry, J. Phys.: Condens. Matter **8**, 1085 (1996).

[7] Ansoft, Ansoft HFSS 8.0 User Documentation, Ansoft Corporation, Pittsburgh, PA, 2001.

[8] D. R. Smith, S. Schultz, P. Markos, and C. M. Soukoulis, Phys. Rev. B **65**, 195104 (2002).

[9] C. G. Parazzoli, Boeing Phantom Works, Seattle, WA.

[10] D. R. Smith, Duke University, Durham, NC.